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Dean Rosenthal <deanrosenthal@gmail.com>

Well-formed scales

6 messages

Dean Rosenthal <deanrosenthal@gmail.com> **Thu, Jun 26, 2008 at 1:56 AM**
To: ncarey@gc.cuny.edu

Dear Norman Carey,

Recently, in a masterclass, I had the opportunity to learn about how to form a well formed scale - from a musician's point of view - I wasn't able to figure out how to do this - 16 points, scales of 7 pitches - somehow the intervals equal, there are two, and there is a pattern as to how to distribute the pitches maximally and correctly.

Could you shed some light on this?

Best.

--
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Norman Carey <ncarey@gc.cuny.edu> **Thu, Jun 26, 2008 at 10:38 AM**
To: Dean Rosenthal <deanrosenthal@gmail.com>

Dear Dean,

Just out of curiosity, where was this masterclass given, and who presented it?

If you want to form a well-formed scale with 7 pitches in within a chromatic scale of 16 equal steps, the simplest way to do this is to apply an algorithm from Clough and Douthett's work on "Maximally Even Sets."

[n•16/7], where n goes from 0 to 6.

That is, take the integral parts (erase the fractions) of the product of 16n over 7

Goes like this:

[0/7] = 0
[16/7] = 2 (16/7 = "2 + 2/7" - just erase the 2/7 and keep 2. That's what the brackets do: [5.312313] = 5, for example.)
[32/7] = 4
[48/7] = 6
[64/7] = 9
[80/7] = 11
[96/7] = 13

So, the scale uses these notes from an equal-tempered 16 note scale:
0, 2, 4, 6, 9, 11, 13.
You can see that the scale has two different step sizes: at places it has two (from 0 to 2, 2 to 4, etc.) and some places it has three (from 6 to 9 and 13 to 16=0).

If you mark an "A" where each smaller interval occurs and a "B" where a larger one appears, we get
0 A 2 A 4 A 6 B 9 A 11 A B

As it turns out, this pattern is exactly the same as the ordinary diatonic scale: AAABAAB. If you start on F and move up by white notes, each A would stand for a whole step, each B by a half step. Not all well-formed scales have exactly this pattern, but all well-formed scale patterns share a lot of traits in common.

Now all of that might be more of a mathematician's point of view. The difficulty of presenting this example from a musician's point of view is that we don't have a strong instinct about intervals that lie outside our standing tunings. However, to take a bit of a stab at it, again look at the pattern AAABAAB. When we let the As = 2 and the Bs = 1, then there are five 2s and two 1s, making 12 in all, which is our usual diatonic scale in equal temperament. If, however, we keep the As at 2 and increase the size of the Bs to 3, then there are five 2s and two 3s, or 16 in all. That pattern can be turned into any number of well-formed scales by letting for each case A take on some value and B take on another. Even if A = B, this too is a well-formed scale, our so-called "degenerate" well-formed scale. There are an infinite number of these patterns, our familiar AAABAAB only one of them.

The work that David Clampitt and I did on well-formed scales includes a wider range of possibilities than the demonstration above might suggest. We do not make any requirements about equal-temperament, for example. Our first paper on well-formed scales used the Pythagorean tuning of the diatonic scale as a starting point, which is not to be found in any equal temperament. Thus, the As and Bs can also take on fractional and even irrational values.

Best wishes,

Norman Carey

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Dean Rosenthal <deanrosenthal@gmail.com> **Thu, Jun 26, 2008 at 10:41 AM**
To: lewis.krauthamer@yahoo.com

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Dean Rosenthal <deanrosenthal@gmail.com> **Thu, Jun 26, 2008 at 3:13 PM**
To: Norman Carey <ncarey@gc.cuny.edu>
Cc: Tom Johnson <tom@johnson.org>

Dear Norman,

Well, I'll tell you, we're in Germany, and the Meisterklassen being given is by the American composer Tom Johnson, based in Paris. The course now taking place is being given in Karlsruhe, a small city, at the Hochshule für Musik.

I hope that helps.

All the very best,
Dean

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Dean Rosenthal <deanrosenthal@gmail.com> **Wed, Jul 9, 2008 at 7:34 AM**
To: Norman Carey <ncarey@gc.cuny.edu>

Dear Prof. Carey,

I write to thank you for your earlier email, the one below, for your helpful input so far. The terms you used and the examples you gave were clear and direct, and I appreciate such a thorough reply! And it came the same day!

I have a few new questions, mostly of method. Let us say we want to calculate a wfs of 7 tones in Z12 (for the sake of an easy example) where each point 's given value is a semitone in equal temperament.

I calculate, using the formula you give: 0, 1, 3, 5, 6, 8, 10, or ABBABB.

0/7=0
12/7=1
24/7=3
36/7=5
...
72/7=10

If this is not correct, I am doing something wrong. Similarly, I calculated 7 tones/steps in Z24: 0, 3, 6, 10, 13, 17, 20 or AABABA. Each scale conforms to Myhill's property.

It seems to me that a 'degenerate' wfs is simply a scale where all the intervals are identical, for example, whole tone, chromatic...in toher temperaments, this is...equal. I am, of course, thinking, in equal temperament tuning, which is the tuning that generally and usefully interests me, right now.

What if I want to form a wfs in some finite set with a particular 'generator' - say of a perfect fifth in a finite set of 12, Z12, simply?

Best regards,
Dean

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Carey, Norman <NCarey@gc.cuny.edu> **Wed, Jul 9, 2008 at 11:08 AM**
To: Dean Rosenthal <deanrosenthal@gmail.com>

>Dear Prof. Carey,
>
>I write to thank you for your earlier email, the one below, for your helpful input so far.
>The terms you used and the examples you gave were clear and direct, and I appreciate
> such a thorough reply! And it came the same day!
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>24/7=3
>36/7=5
>...
>72/7=10
>
>If this is not correct, I am doing something wrong.

Almost: The As and Bs represent the two different size steps, but there is a final step - B - from 10 back "up" to 0 = 12.
So the pattern of steps should include all seven: ABBABBB.

>Similarly, I calculated 7 tones/steps in
>Z24: 0, 3, 6, 10, 13, 17, 20 or AABABA. Each scale conforms to Myhill's property.
>
Again, you would need to complete the representation with a final 'B', which is the step of size 4 from 20 back "up" to 0 = 24.
This gives AABABAB.

>It seems to me that a 'degenerate' wfs is simply a scale where all the intervals are
> identical, for example, whole tone, chromatic...in toher temperaments, this is...equal. I
> am, of course, thinking, in equal temperament tuning, which is the tuning that generally
> and usefully interests me, right now.

This is completely correct. The degenerate well-formed scale is the final scale in a hierarchy generated by some rational division of the octave. Given the fifth of equal temperament, the well-formed scales have are these: 1, 2, 3, 5, 7, and 12. The 5-note scale is the black-key pentatonic, and the 7 is the diatonic in equal temperament. The final 12-tone scale is is the

complete equal-tempered octave.
>
>What if I want to form a wfs in some finite set with a particular 'generator' - say of a
>perfect fifth in a finite set of 12, Z12, simply?
>
I think that the answer above begins to get at your question: those six scales are well-formed.

Let's build them starting of F:

1 - F (F)
2 - F C (F)
3 - F G C (F)
5 - F G A C D (F)
7 - F G A B C D E (F)
12 - F F# G G# A A# B C C# D D# E (F)

Each of these scales is derived by generating a stack of fifths and then reordering into a single octave. Thus, the 5-note scale can be generated by the notes F C G D A, and put into scalar order as above.

You can verify that all of these, save the last, have Myhill's Property (you can also safely toss out the 1-note scale given its triviality). You can also see that any other scale generated in this way will not have Myhill's Property.

Try stacking 4 fifths:

F C G D (F)

Reordered:

F G C D (F)

Note the step sizes here: FG = 2, GC = 5, CD = 2, DF = 3. So there are three, not two different step sizes.

In this very simplified presentation, there doesn't seem to be any clear way to predict which stacks of fifths will produce well-formed scales. Here, the numbers are small enough that you can use a "brute strength" approach like I just did, but that isn't too informative. You can get a bit more insight into how these scales can be precisely determined by taking a look at them in their ordering by fifths:

1 - F (F)
2 - F C (F)
3 - F C G (F)
5 = F C G D A (F)
7 = F C G D A E B (F)
12- F C G D A E B F# C# G# D# A# (F)

All of the consecutive intervals here are perfect fifths - except for the final one that brings you back to F. What we are able to say in some very well-defined sense is that these intervals are close approximations for the generating perfect fifth. In the case of 5, that final interval spans from A up to F; in the case of 7, from B up to F. Neither of these is an actual perfect fifths, but both are a lot better than the final interval in the case of 6:

6 - F C G D A E (F)

E up to F is clearly a much worse approximation of the perfect fifth than either A to F (pentatonic) or B to F (diatonic). Note that for the 12-note scale, the final interval is A# to F, which is identical to a perfect fifth - if you assume equal temperament.

The mathematics of continued fractions deals with these kinds of questions of approximation. Our first article, Aspects of well-formed scales (in Music Theory Spectrum, v. 11), goes into pretty clear detail about this in the second half of the paper, after introducing these points in a more informal way in the first half.

Best wishes,

Norman Carey